Category Theory Pset 4

October 16, 2018

1. Consider a category $\mathcal{C}$ with all pullbacks and a morphism $f \in \mathcal{C}(a, b)$. Define a functor $f^* \colon \mathcal{C}/b \to \mathcal{C}/a$ which on objects takes $c \to b$ in $\mathcal{C}/b$ to its pullback along f. Prove that f^* is a right adjoint.

2. Let $\mathcal{C}$ be a full, reflective subcategory of $\mathcal{D}$ with $i \colon \mathcal{C} \hookrightarrow \mathcal{D}$ the inclusion. Let J be a small category and $F \colon J \to \mathcal{C}$ a functor such that iF has a limit in $\mathcal{D}$. Show that F has a limit in $\mathcal{C}$. (NB: This is trickier than you might think.)

3. 4.1.ii

4. 4.1.iii

5. 4.1.v

6. 4.2.i (read §4.2 first, and we will review the section in class)